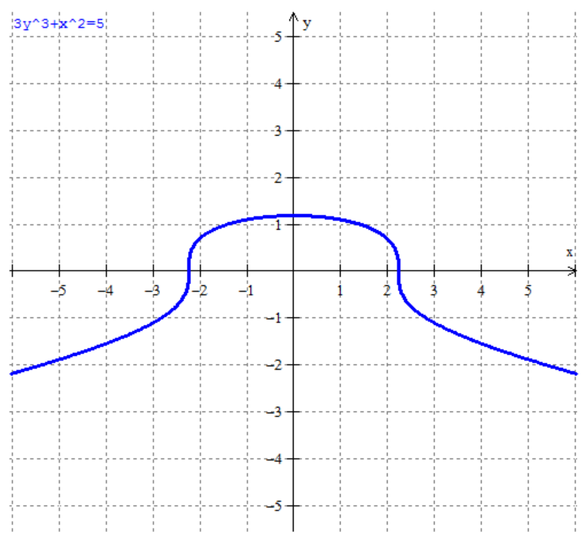
Name:_________________________________ Date:________________________ Band:________

Calculus | Packer Collegiate Institute

Implicit Differentiation, Visually

9. The relation $3y^3 + x^2 = 5$, when graphed is below. And we calculated the derivative as:

$$y' = -\frac{2x}{9y^2}$$



(a) Look at the equation for the derivative. What must be true about the x or y coordinates for the derivative to be undefined?

(b) Now look at the graph. Draw a dot on the point(s) where the derivative is undefined. Write down the coordinates (approximate!). Does your conclusion match what you noticed in part (a)?

(c) Look at the equation for the derivative. What must be true about the x or y coordinates, for the derivative to be 0?

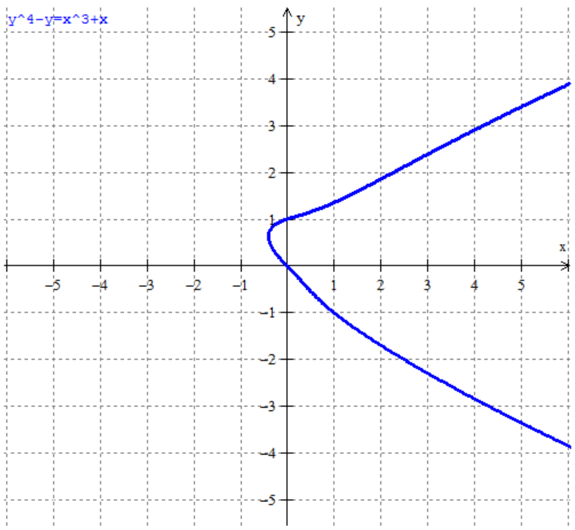
(d) Now look at the graph. Draw a small triangle on the point(s) where the derivative is 0. Write down the coordinates (approximate!). Does your conclusion match what you noticed in part (c)?

(e) Look at the equation for the derivative. When *x* is negative, is the derivative positive or negative? How do you know? What about when *x* is positive – is the derivative positive or negative?

(f) Now look at the graph. Explain how your conclusion in (e) matches the graph.

10. The relation $y^4 - y = x^3 + x$, when graphed is below. And we calculated the derivative as:

$$y' = \frac{3x^2 + 1}{4y^3 - 1}$$



(a) Using the equation for the derivative, show that there are *no* points where the tangent line is horizontal.

(b) Using any method you can, find two points on the original relation $y^4 - y = x^3 + x$.

Point A: ___________ Point B:________________

(c) The slope of the tangent line at Point A is: ________

The slope of the tangent line at Point B is: ________

(d) The equation for the tangent line at Point A: ___________________________

The equation of the tangent line at Point B: ____________________________

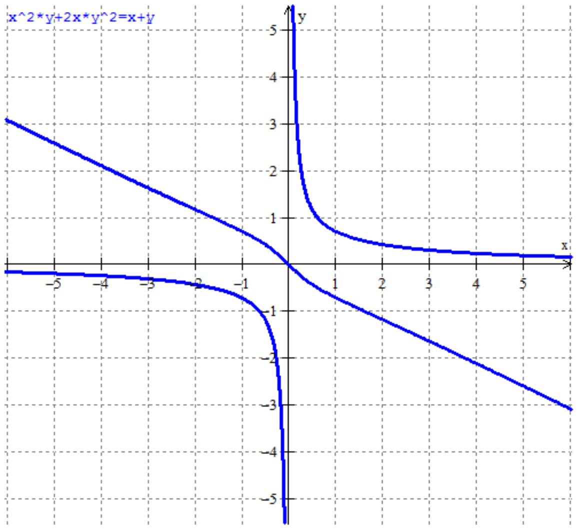
(e) Looking at the graph above, find where the tangent line is vertical. Put a dot there.

(f) Using the equation for the derivative, find the *y*- coordinate of the point where the tangent line is vertical.

(g) Using any method you can, can you find the *x*- coordinate of the point where the tangent line is vertical? (Hint: Your calculator is a great friend!)

11. The relation $x^2 y + 2xy^2 = x + y$, when graphed is below. And we calculated the derivative as:

$$y' = \frac{1 - 2xy - 2y^2}{x^2 + 4xy - 1}$$



(a) I know for a fact that (0,0) is a point on the relation. How do I know that?

(b) You can see at x=1, there are two points on the relation. Put a dot there and approximate the points.

Point A (approx):________________

Point B (approx):________________

(c) If x=1, can you find the *y*- coordinates on the relation exactly? (Hint: Substitute x=1 into the original relation.)

Point A (exact):_______________

Point B (exact):_______________

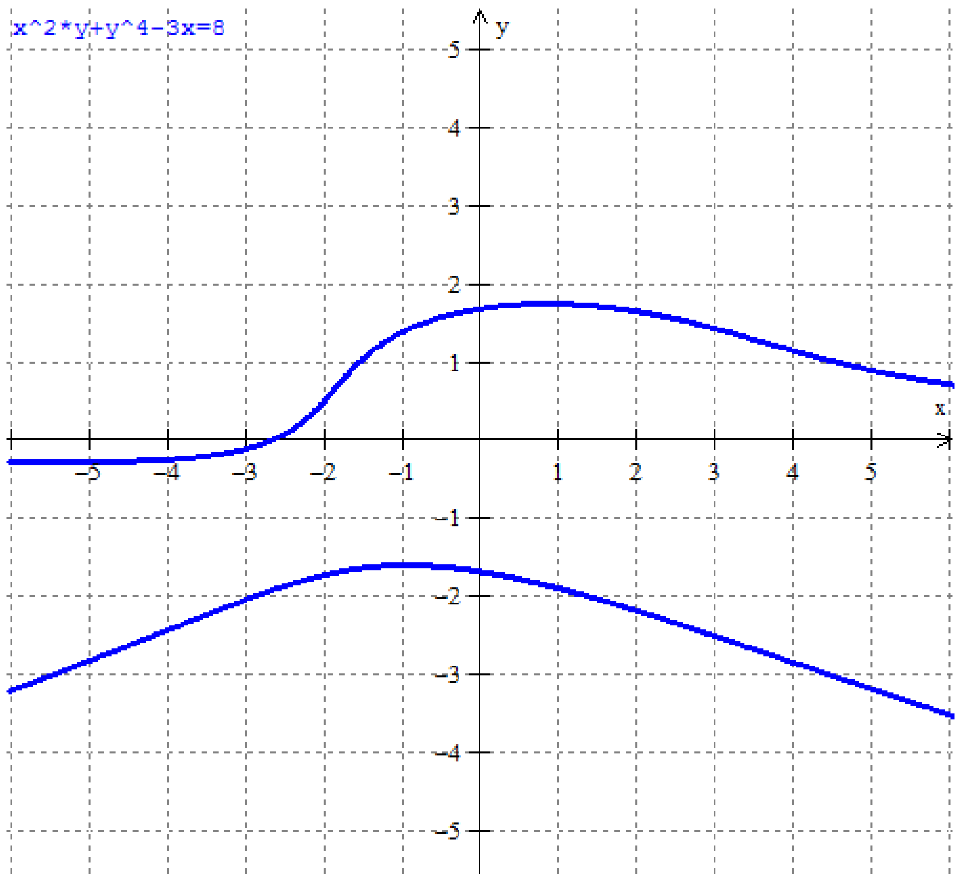
(d) Use your calculators to find the derivative at the two points where x=1.

Derivative at Point A:_____________

Derivative at Point B:_____________

13. The relation $x^2y+y^4-3x=8$, when graphed is below. And we calculated the derivative as:

$$y'=\frac{3-2xy}{x^2+4y^3}$$



(a) Pick any three points on the graph and draw a dot on them. Write what they are approximately:

Point A: ______________

Point B: ______________

Point C:_______________

(b) Now use a straightedge to draw the tangent lines at these three points. Use two points on these lines to estimate as good as you can the slopes. **Round to the hundredths place.**

Slope at Point A (from picture):____________

Slope at Point B (from picture):____________

Slope at Point C (from picture):____________

(c) Now use the points you came up with in Part (a), and the derivative equation we came up with, to come up with the slope of the tangent line at each of the points. **Round to the hundredths place.**

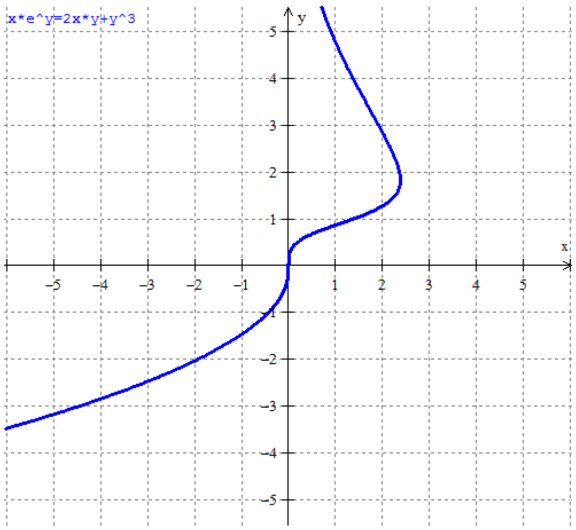
Slope at Point A (from equation):____________

Slope at Point B (from equation):____________

Slope at Point C (from equation):____________

25. The relation $xe^y = 2xy + y^3$, when graphed is below. And we calculated the derivative as:

$$y' = \frac{e^y - 2y}{2x + 3y^2 - xe^y}$$



(a) Solve the relation for *x* totally in terms of *y*.

x=

(b) Find three points on the relation, where the *y* coordinates are:

y=-5, *y*=2, *y*=10

(Round to the hundredths place for x.)

Point A:__________ Point B: __________ Point C: __________

Are these points on the graph?

As you noticed, when *y* was 10, the x-coordinate was really small. What do you think would happen as you make *y* a bigger number, like 100? What would happen to the *x* coordinate? What about an even bigger number, like 1,000?

(c) Now let's look at the derivative. $y' = \frac{e^y - 2y}{2x + 3y^2 - xe^y}$

Find the derivative of Point C (the point with the *y* coordinate of 10). Does it match what the graph suggests? Explain.

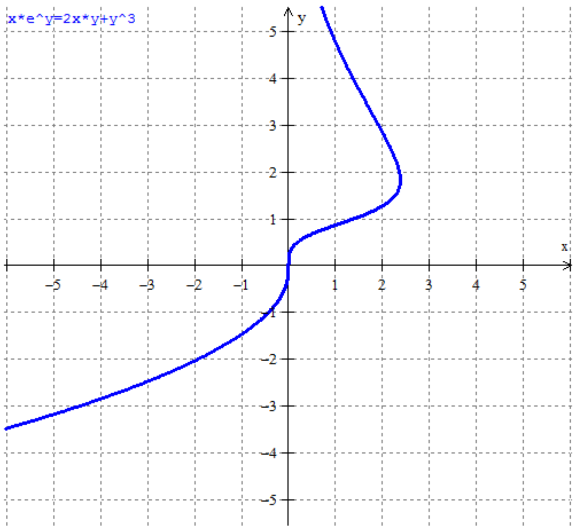
(d) Find the point on the relation which has *y*=100.

Point D:_______________

Find the derivative of Point D (the point with the *y* coordinate of 10). Does it match what the graph suggests? Explain.

25 (super advanced). The relation $xe^y = 2xy + y^3$, when graphed is below. And we calculated the derivative as:

$$y' = \frac{e^y - 2y}{2x + 3y^2 - xe^y}$$



(a) Use any method you want to find some points on the relation. (Hint: choose values for *y* and solve for *x*.)

(Round to the hundredths place for x.)

Point A:__________ Point B: __________ Point C: __________

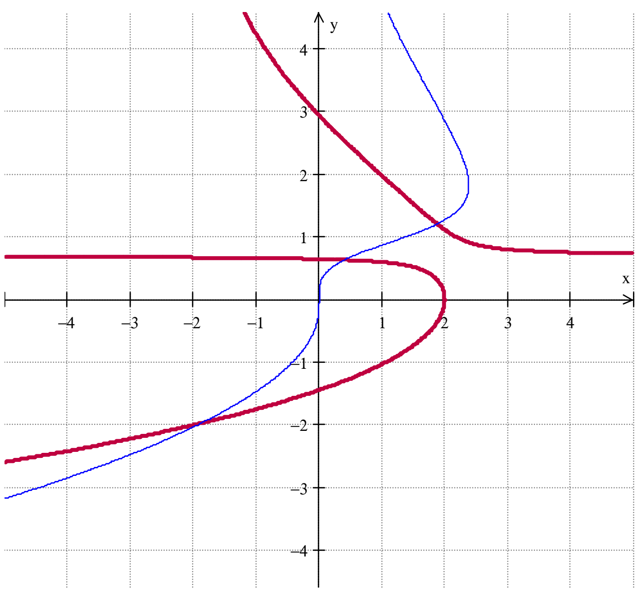
(b) Using the equation for the derivative, find the derivative at the three points:

The derivative at each of these points is:

Point A:__________ Point B: __________ Point C: __________

(c) Look at the graph of the relation above. Does it look like there is any point where the derivative which has value 1/2? If so, mark those points on the graph with a dot!

(d) I am graphing the original relation *and* $\frac{1}{2} = \frac{e^y - 2y}{2x + 3y^2 - xe^y}$



(i) What does the graph for the $\frac{1}{2} = \frac{e^y - 2y}{2x + 3y^2 - xe^y}$ represent? **It is the thicker graph.**

Answer: All (x,y) points where:

____

(ii) What does the graph of the original relation represent? **It is the thinner graph.**

Answer: All (x,y) points where:

____

(iii) What does the intersection points of the graph represent?