Ongoing Skill List for Adv. Precalculus

Skill 1: Do you understand what the graphs of the form $r = \cos(\theta) + k$ and $r = \sin(\theta) + k$ look like (for various values of $k$), and why they look the way they do? Given an equation and a blank graph, can you draw what these graphs would look like?

Skill 2: In terms of graphing in polar, can you explain what certain transformations to the polar equations do? (For example, if you had the graph of $r = \cos(\theta)$, what do the graphs of like $r = \cos(\theta - \pi/6)$, $r = -\cos(\theta)$, and $r = 7\cos(\theta)$ would look like?)

Skill 3: Can you add and subtract vectors graphically? Can you multiply vectors by a scalar and understand what happens graphically?

Skill 4: Can you algebraically add and subtract vectors? Can you algebraically multiply vectors by a scalar? Given the coordinate points that represent the head and tail of a vector, can you calculate what the vector is? Given a vector, can you calculate the magnitude and direction that the vector is pointing? Given a vector, can you come up with another vector that has the same direction but unit magnitude?

Skill 5: Do you know the formula for the dot product between two vectors? Can you use that formula to find the angle between the two vectors? Do you understand what a dot product of 1, 0, and -1 mean in terms of what the two vectors might be? Do you understand what the dot product is conceptually?

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Skill 6: Can you add, subtract, multiply, and divide complex numbers given in rectangular form? Can you plot complex numbers given in rectangular form on a complex plane? Can you show the addition of two complex numbers on a complex plane, and relate it to our work with vectors? [resource(s): Desmos activity “Complex Numbers 1” and delta math]

Skill 7: Can you articulate what multiplying a complex number by a scalar does visually? Can you articulate what multiplying a complex number by an imaginary number does visually? Can you explain visually how multiplication of two complex numbers in rectangular form work? (This is Screen #29 in the desmos activity “Complex Numbers 1.”) [resource(s): Desmos activity “Complex Numbers 1” and delta math]

Skill 8: Given a complex number in rectangular form, can you write it in polar form? Given a complex number in polar form, can you write it in rectangular form? Can you multiply and divide two complex numbers in polar form? [resource(s): Desmos activity “Complex Numbers 2” and delta math]

Skill 9: Can you use DeMoivre’s theorem to raise complex numbers to various powers? Can you identify all solutions to $z^n = 1$ visually and in algebraic form (in both rectangular and polar forms)? Can you relate DeMoivre’s theorem to these solutions to “roots of unity”? [resource(s): Desmos activity “Complex Numbers 2” and delta math]
Skill 10: Can you explain conceptually why a hole appears in a graph (e.g. why a hole appears in \( y = \frac{x + 2}{x + 2} \))? Can you explain conceptually why a vertical asymptote appears in a graph (e.g. why a vertical asymptote appears in \( y = \frac{1}{x} \))? Given a rational equation, can you identify all the x-intercepts, holes, and vertical asymptotes? Can you find the exact location (x- and y-coordinates) of holes? Can you explain how you found the y-coordinate of any holes, and why that approach works? [resource(s): Desmos activity “Rational Functions #1,” Desmos activity “Rational Functions 3,” and delta math]

Skill 11: Given a rational equation, can you identify the end behavior? Given the end-behavior of a rational function and some information about the rational function (e.g. the degree of the numerator, the leading term of the denominator), can you come up with a rational function that would fit the end behavior and information given? Can you explain conceptually why that end behavior results? [resource(s): Desmos activity “Rational Functions #2”]

Skill 12: Given a rational function, can you perform a sign analysis for it? Given a rational function, can you identify all the relevant features of the graph, and then use that information to draw a sketch of the rational function? [resource(s): Desmos activity “Rational Functions #3,” Desmos Activity “Rational Functions #4,” and delta math]

Skill 13: Given an equation for an ellipse that isn’t in standard form, can you write it in standard form? Can you identify the x- and y- intercepts for an ellipse given an equation for an ellipse? Can you identify the foci for an ellipse given the equation for an ellipse? Can you explain why the “string length” for an ellipse is the length of the major axis?